# Moduli Stabilization in String Gas Cosmology
—— *LATEX 2ε Version* ——

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String gas cosmology is an approach towards studying the effects of superstring theory on early universe cosmology which is based on new symmetries and new degrees of freedom of string theory. Within this context, it appears possible to stabilize the moduli which describe the size and shape of the extra spatial dimensions without the need of introducing many extra tools such as warping and fluxes. In this lecture, the recent progress towards moduli stabilization in string gas cosmology is reviewed, and outstanding problems for the scenario are discussed.

## §1. Introduction

In this Introduction, the conceptual problems of scalar field-driven inflationary cosmology (our current paradigm of early universe cosmology) will be reviewed. These are some of the reasons which motivate the search for a new theory of the very early universe. It will then be explained why string theory offers the promise of successfully addressing some of these problems. However, any attempt to combine string theory with cosmology leads to new cosmological problems, most importantly the questions of why only three of the nine spatial dimensions of critical superstring theory are large, and why the moduli fields describing the volume and shape of the extra dimensions are stabilized.

In the context of the current view of cosmology in which the temperature increases without bound as we venture into the past, it is clear that it is the ultimate theory which describes physics at the highest energies and on the smallest scales which will determine the evolution of the universe at the earliest times. At the moment, string theory is the best candidate of providing the required theory of matter at the highest energy scales. Thus, in the following we shall work under the hypothesis that string theory is indeed the correct theory of matter in the very early universe.

However, there are also reasons coming from cosmology alone which might lead us to turn to string theory in the search for the theory of the very early universe: Most implementations of the inflationary universe scenario[1),2)] (see also[3),4)] for earlier ideas), the current paradigm of early universe cosmology, are based on the existence of a slowly rolling scalar field, the *inflaton*, whose energy-momentum tensor is dominated by the contribution of the field potential energy which drives a period of accelerated expansion. In spite of the impressive phenomenological successes of this paradigm in predicting the spectrum of density perturbations and the angular

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power spectrum of cosmic microwave background (CMB) anisotropies, the scalar field-driven inflationary universe scenario suffers from some important conceptual problems.[5),6)] Addressing these problems is another of the goals of superstring cosmology.

The first of the conceptual problems of scalar field-driven inflation is the *amplitude problem*, namely the fact that in the simplest realizations of the model, a hierarchy of scales needs to be present in order to be able to obtain the observed small amplitude of the primordial anisotropies (see e.g.[7)] for a fairly general analysis of this problem).

A more serious problem is the *trans-Planckian problem*.[5)] As can be seen from the space-time diagram of Figure 1, provided that the period of inflation lasted sufficiently long (for GUT scale inflation the number is about 70 e-foldings), then all scales inside the Hubble radius today started out with a physical wavelength smaller than the Planck scale at the beginning of inflation. Now, the current theory of cosmological perturbations (the theory used to calculate the spectra of density fluctuations and microwave anisotropies) is based on Einstein's theory of General Relativity coupled to a simple semi-classical description of matter. It is clear that these building blocks of the theory are inapplicable on scales comparable and smaller than the Planck scale. Thus, the key successful prediction of inflation (the theory of the origin of fluctuations) is based on an incomplete analysis: we know that new physics must enter into a correct computation of the spectrum of cosmological perturbations. The key question is whether the predictions obtained using the current theory are sensitive to the specifics of the unknown theory which takes over on small scales. Toy model calculations using modified dispersion relations[8),9)] have shown that the predictions are in fact sensitive to Planck-scale physics, thus opening up the exciting possibility to test Planck-scale and string-scale physics in current observations (see[10)] for a review with references to other approaches towards exploring this "trans-Planckian window of opportunity").

A further problem is the *singularity problem*. It was known for a long time that standard Big Bang cosmology cannot be the complete story of the early universe because of the initial singularity, a singularity which is unavoidable when basing cosmology on Einstein's field equations in the presence of a matter source obeying the weak energy conditions (see e.g.[11)] for a textbook discussion). Recently, the singularity theorems have been generalized to apply to Einstein gravity coupled to scalar field matter, i.e. to scalar field-driven inflationary cosmology.[12)] It is shown that in this context, a past singularity at some point in space is unavoidable. Thus we know, from the outset, that scalar field-driven inflation cannot be the ultimate theory of the very early universe.

The Achilles heel of scalar field-driven inflationary cosmology is, however, the *cosmological constant problem*. We know from observations that the large quantum vacuum energy of field theories does not gravitate today. However, to obtain a period of inflation one is using the part of the energy-momentum tensor of the scalar field which looks like the vacuum energy. In the absence of a convincing solution of the cosmological constant problem it is unclear whether scalar field-driven inflation is robust, i.e. whether the mechanism which renders the quantum vacuum energy
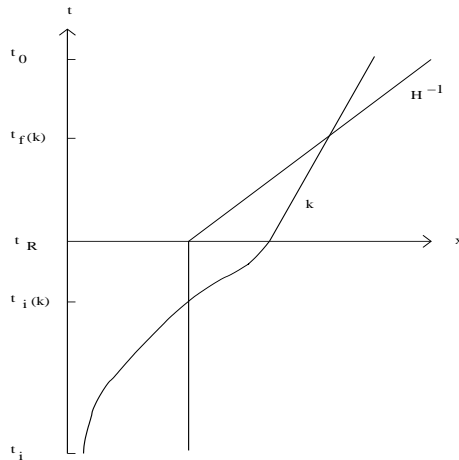
Fig. 1.   Space-time diagram (sketch) showing the evolution of scales in inflationary cosmology. The vertical axis is time, and the period of inflation lasts between $t_i$ and $t_R$, and is followed by the radiation-dominated phase of standard big bang cosmology. During exponential inflation, the Hubble radius $H^{-1}$ is constant in physical spatial coordinates (the horizontal axis), whereas it increases linearly in time after $t_R$. The physical length corresponding to a fixed comoving length scale labelled by its wavenumber $k$ increases exponentially during inflation but increases less fast than the Hubble radius (namely as $t^{1/2}$) after inflation.

gravitationally inert today will not also prevent the vacuum energy from gravitating during the period of slow-rolling of the inflaton field. Note that the approach to addressing the cosmological constant problem making use of the gravitational back-reaction of long range fluctuations (see[13] for a summary of this approach) does not prevent a long period of inflation in the early universe.

Finally, a key challenge for inflationary cosmology is to find a well-motivated candidate for the scalar field which drives inflation, the inflaton. Ever since the failure of the model of *old inflation*,[1,2] it is clear that physics beyond the Standard Model of particle physics must be invoked.

It is likely that string theory will provide ideas which allow us to successfully address some of the above-mentioned problems of the current versions of inflationary cosmology. Foremost, since one of its goals is to resolve space-time singularities, string theory has a good chance of providing a nonsingular cosmology (and *string gas cosmology*, the scenario explored below, indeed has the potential of resolving the cosmological singularity). Since string theory should describe physics on all scales, it should be possible to compute the spectrum of cosmological perturbations in a consistent way within string theory, thus opening the *trans-Planckian window of opportunity*. Finally, string theory contains many light scalar fields, good candidates to be the inflaton.

Immediate problems which arise when trying to connect string theory with cosmology are the *dimensionality* and *moduli problems*. Critical superstring theory is perturbatively consistent only in ten space-time dimensions, but we only see three large spatial dimensions. The original approach to addressing this problem was to assume that the six extra dimensions are compactified on a very small space which

cannot be probed with our available energies. However, from the point of view of cosmology, it is quite unsatisfactory not to be able to understand why it is precisely three dimensions which are not compactified and why the compact dimensions are stable. Brane world cosmology[14] provides another approach to this problem: it assumes that we live on a three-dimensional brane embedded in a large nine-dimensional space. Once again, a cosmologically satisfactory theory should explain why it is likely that we will end up exactly on a three-dimensional brane (for some interesting work addressing this issue see[15]–[17]). In the context of the first approach, one must then understand why the volume and shape of the extra dimensions are stabilized. This is the main aspect of the moduli problem. Any time dependence of the size and shape of the extra dimensions would likely lead to effects on our four space-time dimensional physics such as time-dependent coupling constants or fifth forces which are not observed. In the context of the second approach, a problem which replaces the moduli problem is the question of why gravity is localized precisely on our brane.

Finding a natural solution to the dimensionality and moduli problems are thus key challenges for superstring cosmology. These challenges have various aspects. First, there must be a mechanism which singles out three dimensions as the number of spatial dimensions we live in. Second, the moduli fields which describe the volume and the shape of the unobserved dimensions must be stabilized. As mentioned above, solving the *singularity problem* is another of the main challenges. These are the three problems which *string gas cosmology*[18]–[20] explicitly addresses at the present level of development.

In order to make successful connection with late time cosmology, any approach to string cosmology must also solve the *flatness problem*, namely make sure that the three large spatial dimensions obtain a sufficiently high entropy (size) to explain the current universe. Finally, it must provide a mechanism to produce a nearly scale-invariant spectrum of nearly adiabatic cosmological perturbations.

Since superstring theory leads to many light scalar fields, it is possible that superstring cosmology will provide a convincing realization of inflation (see e.g.[21] for reviews of recent work attempting to obtain inflation in the context of string theory). However, it is also possible that superstring cosmology will provide an alternative to cosmological inflation, maybe along the lines of the Pre-Big-Bang[22] or Ekpyrotic[23] scenarios. The greatest challenge for these alternatives is to solve the flatness problem (see e.g.[24]).

*String gas cosmology* is one approach to combining string theory and cosmology. Its basics will be reviewed in the following section. Recently, there has been a lot of progress on the issue of moduli stabilization in string gas cosmology. This is the main topic of this lecture and will be the focus of the third section.

## §2.  Basics of String Gas Cosmology

In the absence of a non-perturbative formulation of string theory, the approach to string cosmology which we have suggested, *string gas cosmology*[18]–[20] (see also[25] for some original work,[26] for a short review, and[27] for a more comprehensive survey),

is to focus on symmetries and degrees of freedom which are new to string theory (compared to point particle theories) and which will be part of any non-perturbative string theory, and to use them to develop a new cosmology. The symmetry we make use of is *T-duality*, and the new degrees of freedom are *string winding modes.*

We take all spatial directions to be toroidal, with $R$ denoting the radius of the torus. Strings have three types of states: *momentum modes* which represent the center of mass motion of the string, *oscillatory modes* which represent the fluctuations of the strings, and *winding modes* counting the number of times a string wraps the torus. Both oscillatory and winding states are special to strings as opposed to point particles.

The energy of an oscillatory mode is independent of $R$, momentum mode energies are quantized in units of $1/R$, i.e. $E_m = m\frac{1}{R}$, and winding mode energies are quantized in units of $R$, i.e. $E_n = nR$, where both $m$ and $n$ are integers. The T-duality symmetry is a symmetry of the spectrum of string states under the change

$$R \to 1/R \qquad\qquad (2\cdot1)$$

in the radius of the torus (in units of the string length $l_s$). Under such a change, the energy spectrum of string states is invariant if winding and momentum quantum numbers are interchanged $(n,m) \to (m,n)$. The string vertex operators are consistent with this symmetry, and thus T-duality is a symmetry of perturbative string theory. Postulating that T-duality extends to non-perturbative string theory leads[28] to the need of adding D-branes to the list of fundamental objects in string theory. With this addition, T-duality is expected to be a symmetry of non-perturbative string theory. Specifically, T-duality will take a spectrum of stable Type IIA branes and map it into a corresponding spectrum of stable Type IIB branes with identical masses.[29]

We choose the background to be dilaton gravity. It is crucial to include the dilaton in the Lagrangian, firstly since the dilaton arises in string perturbation theory at the same level as the graviton (when calculating to leading order in the string coupling and in $\alpha'$), and secondly because it is only the action of dilaton gravity (rather than the action of Einstein gravity) which is consistent with the T-duality symmetry. Given this background, we consider an ideal gas of matter made up of all fundamental states of string theory, in particular including string winding modes.

Any physical theory requires initial conditions. We assume that the universe starts out small and hot. For simplicity, we take space to be toroidal, with radii in all spatial directions given by the string scale. We assume that the initial energy density is very high, with an effective temperature which is close to the Hagedorn temperature, the maximal temperature of perturbative string theory.

Based on the T-duality symmetry, it was argued[18] that the cosmology resulting from SGC will be non-singular. For example, as the background radius $R$ varies, the physical temperature $T$ will obey the symmetry $T(R) = T(1/R)$ and thus remain non-singular even if $R$ decreases to zero. Similarly, the length $L$ measured by a physical observer will be consistent with the symmetry $(2\cdot1)$, hence realizing the idea of a minimal physical length.[18]

Next, it was argued[18] that in order for spatial sections to become large, the winding modes need to decay. This decay, at least on a background with stable one cycles such as a torus, is only possible if two winding modes meet and annihilate. Since string world sheets have measure zero probability for intersecting in more than four space-time dimensions, winding modes can annihilate only in three spatial dimensions. Numerical confirmation for this scenario was provided in[30] (see also,[31] but see also the recent caveats to this conclusion based on the work of[32]). Thus, only three spatial dimensions can become large, hence explaining the observed dimensionality of space-time. As was shown later,[20] adding branes to the system does not change these conclusions since at later times the strings dominate the cosmological dynamics. Note that in the three dimensions which are becoming large there is a natural mechanism of isotropization as long as some winding modes persist.[33]

The equations of SGC are based on coupling an ideal gas of all string and brane modes, described by an energy density $\rho$ and pressures $p_i$ in the i'th spatial direction, to the background space-time of dilaton gravity. They follow from varying the action

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left[ \hat{R} + 4\partial^\mu \phi \partial_\mu \phi \right] + S_m \,, \tag{2·2}$$

where $g$ is the determinant of the metric, $\hat{R}$ is the Ricci scalar, $\phi$ is the dilaton, $\kappa$ is the reduced gravitational constant in ten dimensions, and $S_m$ denotes the matter action. The metric appearing in the above action is the metric in the string frame. In the case of a homogeneous and isotropic background given by

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \,, \tag{2·3}$$

the three resulting equations (the generalization of the two Friedmann equations plus the equation for the dilaton) in the string frame are[19] (see also[34])

$$- d\dot{\lambda}^2 + \dot{\varphi}^2 = e^\varphi E \tag{2·4}$$

$$\ddot{\lambda} - \dot{\varphi}\dot{\lambda} = \frac{1}{2} e^\varphi P \tag{2·5}$$

$$\ddot{\varphi} - d\dot{\lambda}^2 = \frac{1}{2} e^\varphi E \,, \tag{2·6}$$

where $E$ and $P$ denote the total energy and pressure, respectively, $d$ is the number of spatial dimensions, and we have introduced the logarithm of the scale factor

$$\lambda(t) = \log(a(t)) \tag{2·7}$$

and the rescaled dilaton

$$\varphi = 2\phi - d\lambda \,. \tag{2·8}$$

For the moment, let us focus on the effects of winding and momentum modes individually on the evolution of a homogeneous space-time of $d = 9$ spatial dimensions. In accordance with the ideal gas approximation, we assume that each spatial dimension has the same number of momentum and winding modes. The contribution of the momentum modes to the pressure is positive, with an equation of state

$$P = \frac{1}{d} E \,. \tag{2·9}$$

In contrast, winding modes lead to tension, i.e. negative pressure. The equation of state of winding modes is

$$P = -\frac{1}{d}E \,. \tag{2·10}$$

Since the energy of winding modes scales as $R$, whereas that of the momentum modes is proportional to $1/R$, we see immediately that, as long as the number of momentum and winding modes does not change, the momentum modes dominate for small values of $R$, whereas the winding modes dominate for large values. If the numbers of winding and momentum modes are equal (i.e. $n = m$), then the critical radius, the radius where the pressure changes sign, is the string scale, in this context called the *self-dual* radius.

We now see immediately from (2·5) that a gas of strings containing both stable winding and momentum modes will lead to the stabilization of the radius of the torus: windings prevent expansion, momenta prevent the contraction. The right hand side of the equation can be interpreted as resulting from a confining potential for the scale factor. Note that this behavior is a consequence of having used dilaton gravity rather than Einstein gravity as the background. The dilaton is evolving at the time when the radius of the torus is at the minimum of its potential.

The above background equations thus demonstrate that, in order for any spatial dimensions to be able to grow large, the winding modes circling this dimension must be able to annihilate. In the case of three spatial dimensions, the interaction of string winding modes can be described in analogy with the interaction of cosmic strings. Two winding strings with opposite orientations can intersect, producing closed loops with vanishing winding as a final state. The equations which describe the energy transfer between winding and non-winding strings are given in analogy to the case of cosmic strings (see e.g.[35)–37)] for reviews). First, we split the energy density in strings into the density in winding strings

$$\rho_w(t) = \nu(t)\mu t^{-2} \,, \tag{2·11}$$

where $\mu$ is the string mass per unit length, and $\nu(t)$ is the number of strings per Hubble volume, and into the density in string loops

$$\rho_l(t) = g(t)e^{-3(\lambda(t)-\lambda(t_0))} \,, \tag{2·12}$$

where $g(t)$ denotes the comoving number density of loops, normalized at a reference time $t_0$. In terms of these variables, the equations describing the loop production from the interaction of two winding strings are[38)]

$$\frac{d\nu}{dt} = 2\nu\big(t^{-1} - H\big) - c'\nu^2 t^{-1} \tag{2·13}$$

$$\frac{dg}{dt} = c'\mu t^{-3}\nu^2 e^{3\big(\lambda(t)-\lambda(t_0)\big)} \tag{2·14}$$

where $c'$ is a constant, which is of order unity for cosmic strings but which depends on the dilaton in the case of fundamental strings.[32)]

The system of equations (2·4, 2·5, 2·6, 2·13, 2·14) was studied in[38)] (see also[39)]). It was verified that the presence of a large initial density of string winding modes

causes any initial expansion of $a(t)$ to come to a halt. Thereafter, $a(t)$ will decrease. The resulting increase in the density of winding strings will lead to rapid loop production, and the number of winding strings will decrease, then allowing $a(t)$ to start expanding again. In,[38] this initial evolution of $a(t)$ was called "loitering". In,[38] the analysis was performed using a constant value of $c'$. Taking into account the dilaton dependence of $c'$, one finds[32] that the annihilation mechanism and resulting liberation of the three large dimensions only works if the initial value of the dilaton is sufficiently large.

## §3.  Moduli Stabilization in String Gas Cosmology

In the following, we shall assume that either the mechanism of[18] for setting in motion the preferential expansion of exactly three spatial dimensions works, or, alternatively, that three dimensions are distinguished from the beginning as being large. In either case, we must address the moduli stabilization problem. We will discuss recent progress on this issue in three steps. First, we study the stabilization of the radion degrees of freedom (the radii of the extra dimensions) in the string frame. Next, we consider the analogous problem in the Einstein frame, and, in a third step, we will consider the stabilization of the shape moduli.

### 3.1.  *Radion Stabilization in the String Frame*

Let us first consider radion stabilization in the string frame.[40] For this purpose, we must generalize our ansatz for the metric to an anisotropic one

$$ds^2 = dt^2 - e^{2\lambda}d\mathbf{x}^2 - e^{2\nu}d\mathbf{y}^2 \,, \tag{3.1}$$

where $\mathbf{x}$ are the coordinates of the three large dimensions and $\mathbf{y}$ the coordinates of the internal dimensions.

The variational equations of motion which follow from the dilaton gravity action (2.2) for the above anisotropic metric are

$$-3\ddot{\lambda} - 3\dot{\lambda}^2 - 6\ddot{\nu} - 6\dot{\nu}^2 + 2\ddot{\phi} = \frac{1}{2}e^{2\phi}\rho \tag{3.2}$$

$$\ddot{\lambda} + 3\dot{\lambda}^2 + 6\dot{\lambda}\dot{\nu} - 2\dot{\lambda}\dot{\phi} = \frac{1}{2}e^{2\phi}p_\lambda \tag{3.3}$$

$$\ddot{\nu} + 6\dot{\nu}^2 + 3\dot{\lambda}\dot{\nu} - 2\dot{\nu}\dot{\phi} = \frac{1}{2}e^{2\phi}p_\nu \tag{3.4}$$

$$-4\ddot{\phi} + 4\dot{\phi}^2 - 12\dot{\lambda}\dot{\phi} - 24\dot{\nu}\dot{\lambda} + 3\ddot{\lambda} + 6\dot{\lambda}^2 + 6\ddot{\nu} + 21\dot{\nu}^2 + 18\dot{\lambda}\dot{\nu} = 0\,. \tag{3.5}$$

where $\rho$ is the energy density and $p_\lambda$ and $p_\nu$ are the pressure densities in the non-compact and compact directions, respectively.

Let us now consider a superposition of several string gases, one with momentum number $M_3$ about the three large dimensions, one with momentum number $M_6$ about the six internal dimensions, and a further one with winding number $N_6$ about the internal dimensions. Note that there are no winding modes about the large dimensions ($N_3 = 0$), either because they have already annihilated by the mechanism

discussed in the previous section, or they were never present in the initial conditions. In this case, the energy $E$ and the total pressures $P_\lambda$ and $P_\nu$ are given by

$$E = \mu\left[3M_3 e^{-\lambda} + 6M_6 e^{-\nu} + 6N_6 e^{\nu}\right] \tag{3.6}$$

$$P_\lambda = \mu M_3 e^{-\lambda} \tag{3.7}$$

$$P_\nu = \mu\left[-n_6 e^{\nu} + M_6 e^{-\nu}\right], \tag{3.8}$$

where $\mu$ is the string mass per unit length. In the following subsection we will consider a more realistic string gas, a gas made up of strings which have momentum, winding and oscillatory quantum numbers together. The states considered here are massive, and would not be expected to dominate the thermodynamical partition function if there are states which are massless. However, for the purpose of studying radion stabilization in the string frame, the use of the above naive string gas is sufficient.

We are interested in the symmetric case $M_6 = N_6$ In this case, it follows from (3.8) that the equation of motion for $\nu$ is a damped oscillator equation, with the minimum of the effective potential corresponding to the self-dual radius. The damping is due to the expansion of the three large dimensions (the expansion of the three large dimensions is driven by the pressure from the momentum modes $N_3$). Thus, we see that the naive intuition that the competition of winding and momentum modes about the compact directions stabilizes the radion degrees of freedom at the self-dual radius generalizes to this anisotropic setting.

### 3.2. *Radion Stabilization in the Einstein Frame using Massless Modes*

In order to make contact with late time cosmology, it is important to consider the issue of radion stabilization when the dilaton is frozen, or, more generally, in the Einstein frame. As was discussed in[41),42)] (see also earlier comments in[40)]), the existence of string modes which are massless at the self-dual radius is crucial in obtaining radion stabilization in the Einstein frame (for more general studies of the importance of massless modes in string cosmologys see[43),44)]). Such massless modes do not exist in all known string theories. They exist in the Heterotic theory, but not in Type II theories.[28)]

Let us consider the equations of motion which arise from coupling the Einstein action (as opposed to the dilaton gravity action) to a string gas. In the anisotropic setting when the metric is taken to be

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 - \sum_{\alpha=1}^{6} b_\alpha(t)^2 dy_\alpha^2, \tag{3.9}$$

where the $y_\alpha$ are the internal coordinates, the equation of motion for $b_\alpha$ becomes

$$\ddot{b}_\alpha + \left(3H + \sum_{\beta=1,\beta\neq\alpha}^{6} \frac{\dot{b}_\beta}{b_\beta}\right)\dot{b}_\alpha = \sum_{n,m} 8\pi G \frac{\mu_{m,n}}{\sqrt{g}\epsilon_{m,n}}\mathcal{S} \tag{3.10}$$

where $\mu_{m,n}$ is the number density of $(m,n)$ strings, $\epsilon_{m,n}$ is the energy of an individual $(m,n)$ string, and $g$ is the determinant of the metric. The source term $\mathcal{S}$ depends

on the quantum numbers of the string gas, and the sum runs over all momentum numbers and winding number vectors $m$ and $n$, respectively (note that $n$ and $m$ are six-vectors, one component for each internal dimension). If the number of right-moving oscillator modes is given by $N$, then the source term for fixed $m$ and $n$ is

$$\mathcal{S} = \sum_\alpha \big(\frac{m_\alpha}{b_\alpha}\big)^2 - \sum_\alpha n_\alpha^2 b_\alpha^2 + \frac{2}{D-1}\big[(n,n) + (n,m) + 2(N-1)\big]. \qquad (3\cdot11)$$

To obtain this equation, we have made use of the mass spectrum of string states and of the level matching conditions. In the case of the bosonic superstring, the mass spectrum for fixed $m, n, N$ and $\tilde{N}$, where $\tilde{N}$ is the number of left-moving oscillator states, on a six-dimensional torus whose radii are given by $b_\alpha$ is

$$m^2 = \frac{m_\alpha}{b_\alpha}\big)^2 - \sum_\alpha n_\alpha^2 b_\alpha^2 + 2(N + \tilde{N} - 2), \qquad (3\cdot12)$$

and the level matching condition reads

$$\tilde{N} = (n,m) + N, \qquad (3\cdot13)$$

where $(n,m)$ indicates the scalar product of $n$ and $m$ in the trivial trivial metric.

There are modes which are massless at the self-dual radius $b_\alpha = 1$. One such mode is the graviton with $n = m = 0$ and $N = 1$. The modes of interest to us are modes which contain winding and momentum, namely

- $N = 1$, $(m,m) = 1$, $(m,n) = -1$ and $(n,n) = 1$;
- $N = 0$, $(m,m) = 1$, $(m,n) = 1$ and $(n,n) = 1$;
- $N = 0$ $(m,m) = 2$, $(m,n) = 0$ and $(n,n) = 2$.

Note that some of these modes survive in the Heterotic string theory, but do not survive the GSO[28] truncation in Type II string theories.

In string theories which admit massless states (i.e. states which are massless at the self-dual radius), these states will dominate the initial partition function. The background dynamics will then also be dominated by these states. To understand the effect of these strings, consider the equation of motion (3·10) with the source term (3·11). The first two terms in the source term correspond to an effective potential with a stable minimum at the self-dual radius. However, if the third term in the source does not vanish at the self-dual radius, it will lead to a positive potential which causes the radion to increase. Thus, a condition for the stabilization of $b_\alpha$ at the self-dual radius is that the third term in (3·11) vanishes at the self-dual radius. This is the case if and only if the string state is a massless mode.

The massless modes have other nice features which are explored in detail in.[42] They act as radiation from the point of view of our three large dimensions and hence do not lead to a over-abundance problem. As our three spatial dimensions grow, the potential which confines the radion becomes shallower. However, rather surprisingly, it turns out the the potential remains steep enough to avoid fifth force constraints.

Key to the success in simultaneously avoiding the moduli overclosure problem and evading fifth force constraints is the fact that the stabilization mechanism is an intrinsically stringy one, as opposed to an effective field theory mechanism. In

the case of effective field theory, both the confining force and the overdensity in the moduli field scale as $V(\varphi)$, where $V(\varphi)$ is the potential energy density of the field $\varphi$. In contrast, in the case of stabilization by means of massless string modes, the energy density in the string modes (from the point of view of our three large dimensions) scales as $p_3$, whereas the confining force scales as $p^{-1}$, where $p_3$ is the momentum in the three large dimensions. Thus, for small values of $p$, one simultaneously gets large confining force (thus satisfying the fifth force constraints) and small energy density.[42),52)]

### 3.3. *Shape Moduli Stabilization*

In the presence of massless string states, the shape moduli also can be stabilized, at least in the simple toroidal backgrounds considered so far[45)] (see also,[46)] a paper which appeared after the Kyoto workshop). To study this issue, we consider a metric of the form

$$ds^2 = dt^2 - d\mathbf{x}^2 - G_{mn}dy^m dy^n \,, \tag{3·14}$$

where the metric of the internal space (here for simplicity considered to be a two-dimensional torus) contains a shape modulus, the angle between the two cycles of the torus:

$$G_{11} = G_{22} = 1 \tag{3·15}$$

and

$$G_{12} = G_{21} = \sin\theta \,, \tag{3·16}$$

where $\theta = 0$ corresponds to a rectangular torus. The ratio between the two toroidal radii is a second shape modulus. However, from the discussion of the previous subsection we already know that each radion individually is stabilized at the self-dual radius. Thus, the shape modulus corresponding to the ratio of the toroidal radii is fixed, and the angle is the only shape modulus which is yet to be considered.

Combining the 00 and the 12 Einstein equations, we obtain a harmonic oscillator equation for $\theta$ with $\theta = 0$ as the stable fixed point.

$$\ddot{\theta} + 8K^{-1/2}e^{-2\phi}\theta = 0 \,, \tag{3·17}$$

where $K$ is a constant whose value depends on the quantum numbers of the string gas. In the case of an expanding three-dimensional space we would have obtained an additional damping term in the above equation of motion.

We thus conclude that the shape modulus is dynamically stabilized at a value which maximizes the area to circumference ratio.

### 3.4. *Discussion*

In this section we have discussed the dynamical stabilization of volume and shape moduli in string gas cosmology. We have seen that simple but stringy mechanisms (not visible from an effective field theory point of view) yield a robust stabilization mechanism for both volume and shape moduli. The mechanism relies in a crucial way on the existence of massless string modes involving both momentum and winding. Neither fluxes nor non-perturbative effects are required to stabilize these moduli

fields, rendering the stabilization mechanism much easier to analyze than the stabilization mechanisms invoked in the case of effective field theory flux compactification scenarios.[51] The stabilization mechanism is more stringy than the mechanisms used in the work of[51] and followup papers.

In string gas cosmology models based on a Heterotic string theory (which has the crucial massless modes), the dilaton is not yet stabilized in the analyses performed to date (see e.g.[47]–[50] for papers discussing some of the challenges of dilaton stabilization). It appears that new ingredients are required. In,[52] ideas for dilaton stabilization using fundamental D-branes were put forward. However, D-branes do not exist in Heterotic theories. In models where the dilaton can be simply stabilized using string/brane gas cosmology ideas, the radion cannot. More work is thus required to convincingly fix all moduli in string gas cosmology.

The results on moduli stabilization in string gas cosmology have been reached by use of simple stringy ingredients. This is to be contrasted with the more involved concepts which must be used in order to stabilize the moduli in the context of low energy effective supergravity theories[51] (see e.g.[53] for a review with a comprehensive list of references). On the other hand, the techniques have only been demonstrated for simple toroidal backgrounds (for extensions to certain orbifolds see[54]).

## §4.   Challenges for String Gas Cosmology

There are significant outstanding challenges for string gas cosmology. First of all, there are basic issues of consistency to be addressed. At the self-dual radius, higher order corrections to the background equations will become important. It should be checked whether the main conclusions reached to date in string gas cosmology are robust towards these corrections. At first sight, the answer should be "yes", since the main conclusions depend on new degrees of freedom and on new symmetries which are valid beyond perturbative string theory.

A challenge already mentioned in the previous section is to complete the program of stabilizing all moduli fields. In the case of models based on Heterotic string theory, the dilaton remains to be fixed. In models based on Type II string theory, the volume modulus is the key modulus which remains to be stabilized.

An outstanding problem for string gas cosmology is to establish a successful connection with late time cosmology. One avenue would be to have a period of inflation of the three large dimensions (for ideas on how to obtain this see[55]–[57]). However, the danger in such a scenario is that the number density of strings winding the extra dimensions is diluted to an extent inconsistent with moduli stabilization. Thus, starting from initial conditions where all spatial dimensions are small, the challenge is to obtain inflation consistent with moduli stabilization. Obviously, it is possible that our three spatial dimensions were always sufficiently large, even when the density of strings winding the extra dimensions was large. Novel ideas for late-time cosmology in this context were recently put forwards in.[58],[59]

It is possible that string gas cosmology will connect with the late time universe without a period of conventional inflation. In this case, a new mechanism for the origin of cosmological fluctuations would be required. A key challenge in this context

is to find a solution of the entropy problem, namely to explain why our three spatial dimensions are so large.[24]

In conclusion, we have shown that string gas cosmology provides the potential for yielding a non-singular cosmology, a cosmology which explains why only three spatial dimensions are very large, and why the moduli describing the volume and shape of the extra dimensions are stabilized. Much more work remains to be done to put string gas cosmology on a more consistent mathematical basis (see[60] for some recent papers on string gas cosmology which were not explicitly discussed in this brief review).

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